The Results of Superposition of Different Actions Upon the Mechanical Structures and Living Bodies

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At present, the analysis of the cumulative effects of different actions (mechanical, thermal, electrical, magnetic, chemical a.s.o.) upon a engineering structure, can be done generally for linear matter behavior. A calculation mode, based on nonlinear law of matter behavior, is proposed for the calculus of the structures lifetime, as well as of the living organisms minimum time of medical treatment. The paper addresses the following issues: the influence of matter deterioration on lifetime and on time of medical treatment; the unitary calculation mode for the lifetime of engineering structures and minimum time of medical treatment for living organisms, based on the matter behavior under the stresses it has been submitted to. The numerical examples allowed the understanding of the way lifetime is induced and the minimum time of medical treatment is influenced.

Keywords: engineering structure lifetime, time of medical treatment, nonlinear behavior, fatigue, effects superposition.

Lifetime is one of the important problems of engineering structure. A wealth of research has been carried out, regarding lifetime prediction under multiaxial loading [1 - 5], creep – fatigue [6; 7], fatigue in the case of fatigue crack growth [8 - 15].

At present, the lifetime extension of certain industrial structures while meeting the safety conditions has come to be considered an important issue: the prescribed lifetime for steam turbines is 40 years; the issue of extending it to 60 years is considered.

One of the main tasks at present, is to analyse if is possible to extend by 15 years the lifetime of some of the first generation nuclear reactors (in operation since 1970), whose nominal design service life was 30 years [16].

The nominal design life of the draglines, used in open coal mines around Australia, is approximately 20 years. Because 40% have been in operation more than 20 years and considering the high capital cost of replacement there was increased pressure to extend their operational life [17].

From ancient times man has asked himself about his lifetime. In the case of living organisms, how long is the remaining lifetime after experiencing overstress, fatigue, disease and treatment? Is it possible the prescription of medicines and a precis treatment program – based on mathematical correlations – as to avoid the diminishing of the lifetime?!

Matter deterioration [18] is one of the main cause of lifetime diminution. This finding underlies the concept of a unified treatment of the lifetime problem in engineering structures and living organisms.

The paper draws a parallel between the lifetime of engineering structures and time of medical treatment for living organisms, on the basis of the concept of specific energy participation, a nondimensional variable that makes it possible to work out the algebraic sum of the external effects, independent of their nature (whether mechanical, thermal, electrical, magnetic, chemical etc.), in the case of engineering structures, or the action of viruses, toxins, bacteria, pollutants, noise, stress factors, medicine etc. in the case of living organisms.

The lifetime of engineering structures and living organisms

The lifetime of engineering structures is influenced by the loading they undergo namely mechnical (static, fatigue, shock), thermal (under or over creep temperature), mecanochemical (corrosion, erosion), in a permanent or in transition regime.

External aggressions represented by noise, pollutants, toxins, viruses, bacteria, stress factors, as well as the fatigue and the effect of medicines, of treatments, are considered stresses for living organisms.

Both – engineering structure and living organisms – are characterized by critical stress values or by critical values of the obtained effects. Critical stress is that value of stress that determines the death of the living organism, the taking out of use or the destruction of an engineering structure.

At present one uses some empirical relations to calculate the lifetime for loading under several blocks of mechanical stress namely (table 1):

- under static loading in creep conditions the critical state (fracture) is reached when Robinson's [19] empirically established relation is fulfilled (1);

- under fatigue cyclic loading one uses Palmgren-Miner empirical relation (2), or in the general case of materials with nonlinear behavior the theoretical relation (3).

When evaluating the result of effects superposition of different stresses on the human body, each stress gets assigned a number, n_i; the more dangerous the effect, the higher the assigned number. The sum $n_t = \sum_i n_i$ is the total trauma index. If $n_t \le n_{cr}$ where n_{cr} defines the critical trauma index then, there is no risk of death for the living organism. If $n_t > n_{t,cr}$ then the living organism may die (4).

In a similar manner one proceeds when evaluating the influence of different factors with stress effect over the risk of becoming ill and the possibility of social readaptation [26].

In the situation of organism exposure to electrical fields of frequencies lower than 10 MHz, if the relation (5) is fulfilled it is dangerous for the organism.

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Table 1
NOWADAYS RELATIONS TO THE EFFECT OF SUPERPOSITION OF DIFFERENT LOADINGS

Nr.	Loading	Relation	Observations
1.	Static loading under creep conditions ($T \geq T_c$ – creep temperature)	$\sum_i \frac{t_i}{t_{i,f}} = 1$ (1)	Robinson [19]. t_i – time of loading under stress σ_i; $t_{i,f}$ – value of t_i at failure.
2.	Fatigue cyclic loading	$\sum_i \frac{N_i}{N_{i,f}} = 1$ (2)	Palmgren-Miner [20 - 23]. N_i – the number of loading cycles whose stress amplitude is $\sigma_{a,i}$; $N_{i,f}$ – number of loading cycles with $\sigma_{a,i}$ down to failure.
3.		$\sum_i \left(\frac{N_i}{N_{i,f}}\right)^{\frac{\alpha+1}{m}} = C_D$ (3)	Jinescu [24]. $\alpha = 1/k$ and m are material constants. C_D is a function which depends on the deterioration.
4.	Superposition of different stresses on the human body in surgery.	$\sum_i n_i = n_{cr}$ (4)	– the organ injury scale [25] $n_i = (\text{risk factor}) \times (\text{injury grade})$; n_{cr} – the critical trauma index.
5.	Organisms exposure to electrical fields of a certain frequencies.	$\sum_i \frac{J_i}{J_{i,L}} \leq 1$ (5)	J_i the density $(\mathrm{A \cdot m^{-2}})$ of the electric current induced at frequency i; $J_{i,L}$ – the J_i limit corresponding to maximum allowable effects [27].
6.	More pollutants action on the environment or upon a living organism	$\sum_i \frac{c_i}{c_{i,cr}} = 1$ (6)	c_i – the concentration of the polluting agent i; $c_{i,cr}$ – critical of c_i value for the environment or the living organism.

If more pollutants act simultaneously on the environment or upon a living organism, their cumulative effect which induces the critical state is calculated with the empirical relationship (6).

Superposition of loading effects on the basis of critical energy principle

Principle of critical energy

One considers a body under a number of loads Y_i (i= 1,2,...i...n). As an answer to the loads action one has the effects X_j =(j= 1,2,...i...n...p).

Effect superposition consists in the determination of the total effect X due to the action of several loads Y_i. So are for example the loadings of an engineering structure with different forces, bending moments, twisting moments etc., or the stresses that determine the manifestation of a disease: for example diabetes plus a cardiovascular affection, or simultaneous illnesses of the thyroidal gland and the heart etc. The following question can be raised: what is the value of the total effect X and how is it calculated if the loads Y_i are known?

The nature of loads' effects can be mechanical, thermal, electrical, magnetic, chemical, nuclear, biophysical etc. Through the actual methods one cannot establish prior to the experiment if the load is critical or not!

In the quantitative analysis of loads effects superposition there must be introduced the influence of the load rate, because the effect depends on this.

Also, when cumulating effects, must be taken into account the way these are applied, simultaneously or successively, because the total effect is influenced by this fact. For example what is the best way the medicines should be administered, simultaneously or successively? As to answer to this question, a quantitative theory must be developed.

According to the principle of critical energy [28 - 32], the critical state attained in a process or phenomenon is reached when the sum of the specific energy amounts involved, considering the sense of their action, becomes equal to the value of the specific critical energy characterizing that particular process or phenomenon. The mathematical expression of the principle of critical energy

is, $$\sum_i \left(\frac{E}{E_{i,cr}}\right)_i \cdot \delta_i = 1, \qquad (7)$$

where E_i is the specific energy used in the process (J/m^3, or J/m^2, or J/kg); $E_{i,cr}$ - critical value of E_i. In a given case $E_{i,cr} = E_{cr}$ is a constant independent of the nature or type of energy involved in the process or phenomenon under analysis; δ_1=1; 0 or -1 if E_i acts in the sense, has no effect or opposes the development of the process or the phenomenon under consideration. The nondimensional variable

$$P_i = \left(\frac{E_i}{E_{i,cr}}\right) \cdot \delta_i, \qquad (8)$$

represents the *specific energy participation* of the external load Y_i with respect to its critical state during the process or phenomenon under consideration. Out of relation (7) and (8) the *total specific energy participation with respect to the* matter critical state,

$$P_T = \sum_i P_i . \qquad (9)$$

The critical value of P_T is the critical participation, generally time dependent (*t*), P_{cr}(t). Its maximum value $P_{cr,max}$ (t)=1.0.

P_{cr}(t) depends on time through the deterioration D(t), as follows

$$P_{cr}(t) = 1 - D(t). \qquad (10)$$

Specific energy participation, critical participation and deterioration are dimensionless.

P_T = 1 for perfect materials free of internal stresses, no-preloading etc. The real materials state becomes critical if,

$$P_T = P_{cr}(t), \qquad (11)$$

where $P_{cr}(t) \le 1$. By solving the equation (11) results the lifetime.

At a certain moment *t,* one writes the difference in participations,

$$\Delta P(t) = P_{cr}(t) - P_T(t). \qquad (12)$$

If ΔP(t)>0, out of relation (12) one can calculate *the residual or remaining lifetime.*

By reducing the effect of the deterioration or by reducing the loading at a given moment one can ensure the *extension of the lifetime* [33].

Participation of specific energy vs. matter behavior

In the general case of the nonlinear behavior given by law,

$$Y = C \cdot X^k, \qquad (13)$$

where *C* and *k* are material constants, the specific energy introduced by an action *Y* into a material body upon which effect *X* is produced, is calculated with relation

$$E = \int_0^X Y \cdot dX = \frac{Y^{1/k+1}}{(k+1) \cdot C^{1/k}} . \qquad (14)$$

The value of E_{cr} is obtained by replacing *Y* with its critical value Y_{cr} such as out of relations (8) and (14), in the case of i^{th} loading, the participation of specific energy,

$$P_i = \left(\frac{Y_i}{Y_{i,cr}}\right)^{\alpha+1} \cdot \delta_i, \qquad (15)$$

where α=1 / k. Generally α depends on the value of the loading rate. In a first round of approximation one can write [30; 34]:

$$\alpha = \begin{cases} \frac{1}{n}, & \text{for static loading;} \\ \frac{1}{2n}, & \text{for rapid loading;} \\ 0, & \text{for shock loading.} \end{cases}$$

If the load *Y* produces an effect, *X*, only after exceeding a value Y_o > 0, the behaviour law (13) becomes,

$$Y = Y_0 + C \cdot X^k . \qquad (16)$$

With $\Delta y = y - y_o$ the law (16) writes,

$$\Delta Y = C \cdot X^k . \qquad (17)$$

In this case the participation of specific energy (15) becomes

$$P_i = \left(\frac{\Delta Y_i}{\Delta Y_{i,cr}}\right)^{\alpha+1} \cdot \delta_i, \qquad (18)$$

where $\Delta Y_{i,cr} = (Y_i - Y_0)_{cr}$.

Some cases of superposition of effects

In the case of nonlinear matter behaviour, out of relations (9), (11) and (15), we get the general form of the relation for the principle of critical energy equation,

$$\sum_i \left(\frac{Y_i}{Y_{i,cr}}\right)^{\alpha+1} \cdot \delta_i = P_{cr}(t), \qquad (19)$$

where $Y_{i,cr}$ is a standard value or the value of the critical load at t=0, namely $Y_{i,cr} \equiv Y_{i,cr}(0)$.

Generally, if P_T(t) < P_{cr}(t) – the loading state is subcritical; P_T(t) = P_{cr}(t) – loading has reached the critical state; P_T(t) > P_{cr}(t) – the loading state is super critical.

For *engineering structures* Y_i represents the stresses which influence its lifetime. For example, *under the fatigue cyclic loading* of a mechanical structure, the total participation of specific energy, P_f is a sum of the participation of stress amplitude P(σ_a) and participation of mean stress P(σ_m) :

$$P_f = P(\sigma_a) + P(\sigma_m), \qquad (20)$$

where

$$\sigma_a = 0.5(\sigma_{max} - \sigma_{min}) \text{ and } \sigma_m = 0.5(\sigma_{max} + \sigma_{min}) .$$

In the case of an engineering structure subjected to fatigue under creep conditions the total participation is the sum of fatigue contribution, P_f and the creep contribution, P_c.

One considers a nonlinear behavior of the structure material given by the power law (like (13)),

$$\sigma = M_\sigma \cdot \varepsilon^k, \qquad (21)$$

where M_σ and k are material constants; σ is normal stress and ε – strain.

If the structure is subjected to fatigue, out of relation (15) and (20) result,

$$P_f = \left(\frac{\sigma_a}{\sigma_{-1,s}(N)}\right)^{\alpha+1} + \left(\frac{\sigma_m}{\sigma_u}\right)^{\alpha+1} \cdot \delta_{\sigma_m}, \qquad (22)$$

where $\alpha = 1/k$ and $\delta_{\sigma_m} = \begin{cases} 1, \text{if } \sigma_m > 0; \\ -1, \text{if } \sigma_m < 0; \end{cases}$

$$\sigma_{-1,s}(N) = \frac{\varepsilon_d \cdot \gamma_s}{K_\sigma} \cdot \sigma_{-1}(N). \qquad (23)$$

One uses Basquin's relation [35],

$$, \qquad (24)$$

where the value of the exponent m depends upon the structure material and the domain of the fatigue (Wöhler) curve. Out of relation (24) results,

$$\sigma_{-1}(N) = \sigma_{-1} \cdot \left(\frac{N_0}{N}\right)^{\frac{1}{m}}. \qquad (25)$$

On the other hand the specific energy participation *under creep conditions* [30] is,

$\sigma_a^m \cdot N = \text{constant}$

$$P_c = \left(\frac{t}{t_f}\right)^{\kappa}, \qquad (26)$$

where t is the time under creep conditions; t_f – the value of t at failure, while $\kappa \geq 1$ is a material constant.

On the basis of the principle of critical energy the total participation by cyclic loading under creep conditions is,

$$P_T = P_f + P_c.. \qquad (27)$$

Out of relations (11), (18), (21), (26) and (27) one obtains the fatigue life of an engineering structure under creep conditions,

$$N = N_0 \cdot \left(\frac{\sigma_{-1}}{\sigma_a}\right)^m \cdot \left[P_{cr}(t) - \left(\frac{\sigma_m}{\sigma_u}\right)^{\alpha+1} \cdot \delta_{\sigma_m} - \left(\frac{t}{t_{cr}}\right)^{\kappa}\right]^{\frac{m}{\alpha+1}}. \qquad (28)$$

The case of living organisms

In the case of several actions, Y_p upon a body, the effects engendered will accumulate. An example might be the overall effect of the superposition of actions caused by the simultaneous or sequential application of an energy treatment involving chemical therapy, acupuncture therapy or acupressure etc.

For example [30], a monkey injected with an amount of poliomyelitic virus, $m_{p,cr}$, falls down with poliomielitis, while if injected with amount $m_p < m_{p,cr}$, it will not get ill. If the monkey undergoes stress intensity $S_{p,cr}$, it will die, whereas, if the stress intensity $S < S_{cr}$, the monkey will not die.

One may raise the question: what will happen to the monkey if it undergoes simultaneouly a virus amount $m_p < m_{p,cr}$, and stress intensity $S < S_{cr}$? The virus amount and the stress intensity have diferent units of measurement. They cannot be cumulated.

Let us assume that between the quantity of polimyelitic virus, m_p and its effect upon the body, X_p, there exists a non-linear power function relation (13)written as:

$$m_p = C_p \cdot X_p^{k_p}, \qquad (29)$$

where C_p and k_p are constant.

At the same time, between the intensity of the stressful action, S, and its effect, X_s, one can accept a power dependent relation, written as:

$$S = C_s \cdot X_s^{k_s}, \qquad (30)$$

where C_s and k_s are constant.

The participations of specific energies of polio action and stress action, respectively, are:

$$P(m_p) = \left(\frac{m_p}{m_{p,cr}}\right)^{\alpha_p+1}; \quad P(S) = \left(\frac{S}{S_{cr}}\right)^{\alpha_s+1}, \qquad (31)$$

where $\alpha_p = 1/k_p$ and $\alpha_s = 1/k_s$ if the action is slow, $\alpha_p = (1/2k_p)+1$ and $\alpha_s = (1/2k_s)+1$ if the polio action and the stress, respectively, is fast and $\alpha_p = \alpha_s = 0$ if the respective actions feature shock.

If m_p and S act simultaneously, the total deterioration produced to the body is calculated by summing up the corresponding participations and we get the total participation,

$$P_T = \left(\frac{m_p}{m_{p,cr}}\right)^{\alpha_p+1} + \left(\frac{S}{S_{cr}}\right)^{\alpha_s+1}. \qquad (32)$$

P_T is compared to $P_{Tcr}(t)$. If $m_{p,cr}$, S_{cr} etc. have individual dependent values (age, antecedents, temporary state of health etc.), that is are customized, then one can accept $P_{cr}(t) = 1$.

If, for instance, the polio virus acts slowly and the stress acts by shock, the previous relation is written as:

$$P_T = \left(\frac{m_p}{m_{p,cr}}\right)^{\frac{1}{k_p}+1} + \frac{S}{S_{cr}}. \qquad (33)$$

The deterioration of the body becomes critical (polio sets in) if the total deterioration is $P_T \geq 1$. If $P_T < 1$. If the critical state is not reached (poliomyelitis does *not* set in).

It is considered that between the action of an amount of drug administered to the body and its effect, X_m, the relationship is nonlinear, depending on power:

$$m_m = C_m \cdot X_m^{k_m} \qquad (34)$$

where C_m and k_m are drug constants.

If one administers drug quantity m_m that opposes the virus action, the drug contribution is calculated by using participation:

$$P_m = \left(\frac{m_m}{m_{m,cr}}\right)^{\alpha_m+1} \cdot \delta_m, \qquad (35)$$

where $m_{m,cr}$ is the critical amount of medication, that is the one that can, by itself alone stop or prevent the manifestation of the disease; $\alpha_m = 1/k_m$ – under the slow action of the drug; $\alpha_m = 1/(2k_m)$ – under rapid action and $\alpha_m = 0$ – under the shock action of the drug.

Because the medicine is opposed to the onset of the disease (acting against deterioration), δ_m = -1 so that, in this case, the total participation is

$$P_T = P(m_p) + P(S) + P_m(m_m),$$

or

$$P_T = \left(\frac{m_p}{m_{p,cr}}\right)^{\alpha_p+1} + \left(\frac{S}{S_{cr}}\right)^{\alpha_s+1} - \left(\frac{m_m}{m_{m,cr}}\right)^{\alpha_m+1} \quad (36)$$

If

$P_T \geq 1$, the critical state is attained or exceeded;

$P_T < 1$, the critical state of the illness and death, respectively, are not attained;

$P_T \to 0$, the body tends to its natural state of equilibrium, corresponding to unaltered health.

The critical variables $m_{p,cr}$, S_{cr} etc. – in living organisms - depend on age. The value of each critical variable decreases over time, so the amount of deterioration – at the same intensity of external action - increases. In old age, for example, the effect of the same virus is greater than in youth etc. The problem, however, may be treated in another way, namely: the values of the critical variables at the initial moment are maintained unaltered (t=0), and $m_{p,cr}(0)$, $S_{cr}(0)$, $m_{m,cr}(0)$, respectively etc., as standard values, while the critical participation is calculated with relation (10) where one introduces the value of body deterioration until the moment of calculation.

In the general case when Y_i represents an action that contributes to the deterioration of a body part, cells, body etc., while m_j is an action administered against deterioration (medicine, ultrasound, electric current etc.), the relation of total participation becomes,

$$P_T = \sum_i \left(\frac{Y_i}{Y_{i,cr}(0)}\right)^{\alpha_i+1} \cdot \delta_i + \sum_j \left(\frac{m_j}{m_{j,cr}(0)}\right)^{\alpha_m+1} \cdot \delta_j \cdot \quad (37)$$

where δ_j = -1 if the medicine *j* opposes any of the Y_i actions.

If deterioration increases over time (aging, illness, accidents ...) it means that, with the passage of time, the value of critical participation decreases $P_{cr}(t)$. As a result, the critical state, at a given time, *t,* is attained at values of the external actions, Y_i, that are lower than at t=0, when $P_{cr}(t)$ = 1 .

In the case of *living organisms*, for instance, if Y_i=c_i is the concentration of pollutant *i*, whose critical concentration is , out of relation (19) we get

$$\sum_i \left(\frac{c_i}{c_{i,cr}}\right)^{\alpha_c+1} = 1, \quad (38)$$

where δ_i=1, $P_{cr}(t)$=1, as in this case D(t)=0. In the particular case where α_c = 0 one obtains the empirical relation (6).

If Y_j = Φ_j is the *radiation flow* (ultraviolet, thermal, neutrons, X rays etc.) whose critical value is $Y_{j,cr}$ = $\Phi_{j,cr}$, with δ_i = 1, one obtains,

$$\sum_j \left(\frac{\Phi_j}{\Phi_{j,cr}}\right)^{\alpha_\Phi+1} = 1. \quad (39)$$

If y_k = S_k is the *stress* produced upon an organism, while $Y_{k,cr}$ = $S_{k,cr}$ is its critical value, then out of relation (15),

$$\sum_k \left(\frac{S_k}{S_{k,cr}}\right)^{\alpha_S} = 1. \quad (40)$$

The total effect of the cumulative action of the pollutans, radiations and stresses upon an organism (or upon a particular cell) may be calculated with the total participation of the speciofic energy (9)

$$P_T = \sum_i \left(\frac{c_i}{c_{i,cr}}\right)^{\alpha_c+1} + \sum_j \left(\frac{\Phi_j}{\Phi_{j,cr}}\right)^{\alpha_\Phi+1} + \sum_k \left(\frac{S_k}{S_{k,cr}}\right)^{\alpha_S+1}. \quad (41)$$

If P_T < 1– the critical state is not attained (death for instance), whereas if $P_T \geq 1$ – the critical state is reached or exceeded (the organism dies)). One can use the same approach if the organism features an insufficient concentration of vitamins, of oligoelements etc.

By writing the total participation as in eq. (32), the critical values depended on the total health and age of the patient.

But one may use unique, standard values for the critical values ($c_{i,cr}$, $\Phi_{j,cr}$, $S_{i,cr}$ etc.) and compare P_T with $P_{cr}(t)$ given by eq. (10), where the deterioration is an individual value of each patient. The patient state becomes critical if,

$$P_T = 1 - D(t), \quad (42)$$

where P_T may be written as,

$$P_T = \sum_i C_{c,i} \cdot c_i^{\alpha_c+1} + \sum_j C_{\Phi,j} \cdot \Phi_j^{\alpha_\Phi+1} + \sum_k C_{S,k} \cdot \Phi_k^{\alpha_k+1}, \quad (43)$$

where $C_{c,i} = (1/c_{i,cr})^{\alpha_c+1}$; $C_{\Phi,i} = (1/\Phi_{j,cr})^{\alpha_\Phi+1}$; $C_{S,k} = (1/S_{k,cr})^{\alpha_k+1}$.

In the case of *living organisms*, the *total destructive participation* with respect to the critical state results from relation (15),

$$P_{T,d} = \sum_i \left(\frac{Y_i}{Y_{i,cr}}\right)^{\alpha_i+1}, \quad (44)$$

where Y_i represents the destructive action (virus, bacteria, electromagnetic waves, sound waves, pollutants etc.), while $Y_{i,cr}$ is the critical value of Y_i, that is the one that by acting alone might result in the death of the organism and δ_i= 1.

The value of $Y_{i,cr} = Y_{i,cr}(t)$ in living organisms, depends on age, namely, it will decrease with the lapse of time because of the body's natural deterioration (ageing). Consequently, the corresponding participation, Y_j / $Y_{i,cr}$, increases with time. If $P_T \geq P_{cr}$ the organism dies. Under natural conditions P_{cr}=1.

In order to help the organism survive, or to get beyond the state of temporary illness, one put the condition,

$$P_{T,d} + R(t) = 1, \quad (45)$$

where R(t) is *the reluctant participation which opposes the deterioration*, R(t)is written as,

$$R(t) = \left(\frac{m_j}{m_{j,cr}}\right)^{\alpha_m+1}, \quad (46)$$

where m_j is the quantity of *j* medication swallowed, while $m_{j,cr}$ is the critical value of m_{jj}, that is the maximum amount of medication which is accepted by the living organism.

One considers the drug medium absorption speed $v_j = dm_j/dt$, where from the minimum duration of drug treatment m_j is,

$$t_{j,\min} = \frac{m_j}{v_j}. \quad (47)$$

Out of relations (34) ÷ (36) in the case of a single drug medication results the *minimum treatment duration required* to annihilate the destructive effect,

$$t_{j,min} = \frac{m_{j,cr}}{v_m}\cdot\left(1-P_{T,d}\right)^{\frac{1}{\alpha_m+1}}. \qquad (48)$$

If the destructive action has an effect only if $y_i > y_{i,o}$, taking into account the law (17), one writes,

$$P'_{T,d} = \sum_i \left(\frac{\Delta Y_i}{\Delta Y_{i,cr}}\right)_i^{\alpha_i+1}, \qquad (49)$$

where $\Delta Y_i = Y_i - Y_{i,0}$ and $\Delta Y_{i,cr} = \left(Y_i - Y_{i,0}\right)_{cr}$. In the same time if the drug has an effect only if $m_j > m_{j,0}$ than instead of eq. (35) one use the eq.,

$$R'(t) = \sum_j \left(\frac{\Delta m_j}{\Delta m_{j,cr}}\right)^{\alpha_m+1}, \qquad (50)$$

where $\Delta m_j = m_j - m_{j,0}$ and $\Delta m_{j,cr} = \left(m_j - m_{j,0}\right)_{cr}$.

In consequence *the minimum treatment duration* with the drug *j*, becomes,

$$t_{j,min} = \frac{\Delta m_{j,cr}}{v_j}\cdot\left(1-P'_{T,d}\right)^{\frac{1}{\alpha_m+1}}. \qquad (51)$$

Applications

One considers the case of three loadings (*i* = 1; 2; 3) which act – at the room temperature – in the sense of the deterioration of a crackless *engineering structure* (δ_i=1): – a fatigue loading, P_1; – two constant loadings characterised by the following participations of the specific energy,

$$P_2 = \left(\frac{Y_2}{Y_{2,cr}}\right)^{\alpha_2+1}\cdot\delta_2; \qquad P_3 = \left(\frac{Y_3}{Y_{3,cr}}\right)^{\alpha_3+1}\cdot\delta_3. \qquad (52)$$

$\alpha_2 = 1/k_2$ and $\alpha_3 = 1/k_3$ are the exponents from relation (21) corresponding to loads Y_2 and Y_3. Fatigue life is obtained by using the relations (19); (22); (25) and (52) where one replaces P_1, P_2, P_3 from relations (22) and (52) with $\delta_1=\delta_2=\delta_3=1$(all loads act towards attaining the critical state),

$$N = N_0\cdot\left(\frac{\sigma_{-1}}{\sigma_a}\right)^m\cdot\left[P_{cr}(t) - \left(\frac{Y_2}{Y_{2,cr}}\right)^{\alpha_2+1} - \left(\frac{Y_3}{Y_{3,cr}}\right)^{\alpha_3+1}\right]^{\frac{m}{\alpha+1}}. \qquad (53)$$

In the case of a steel structure fatigue loaded: $m = 3$; $\alpha = \alpha_2 = \alpha_3 = 2.5$; $N_0 = 2\times10^6$. The structure has been deteriored by preloading with $D_T(t) = 0.1$ such as $P_{cr}(t) = 1 - D_T(t) = 0.9$. The loads are defined through the values of the following ratios: $\frac{\sigma_a}{\sigma_{-1}} = 1.6$; $\frac{Y_2}{Y_{2,cr}} = 0.4$; and $\frac{Y_3}{Y_{3,cr}} = 0.5$. By using relation (53) one gets fatigue life,

$$N = 2\times10^6\times\left(\frac{1}{1.6}\right)^3\cdot\left[0.9 - (0.4)^{3.5} - (0.5)^{3.5}\right]^{\frac{3}{3.5}} \approx 3.9\times10^5 \text{ cycles}$$

which is less N_o = 2 x 10^6 cycles.

In the case of cracked structures the critical loadings must be calculated taking into account the crack [36].

One considers two destructive loadings, Y_1 and Y_2, applied to a *living body*. The organism is treated with medication which oppose the organism deterioration. The destructive loads are caracterised by the following rations: $\frac{Y_1}{Y_{1,cr}} = 0.8$ and $\frac{Y_2}{Y_{2,cr}} = 0.6$. The medication is characterised by the ratio $\frac{m_3}{m_{3,cr}}$.

The organism behaviour in contact with loads Y_1, Y_2 and Y_3 is characterized by α_1=2, α_2=1 and α_3=0.5. The participation of the specific energy corresponding to living organism deterioration $P_1 = \left(\frac{Y_1}{Y_{1,cr}}\right)^{\alpha_1+1}$; $P_2 = \left(\frac{Y_2}{Y_{2,cr}}\right)^{\alpha_2+1}$, while the participation of the medication Y_3 is $P_3 = \left(\frac{m_3}{m_{3,cr}}\right)^{\alpha_3+1}$.

One considers load Y_1 featuring shock nature, Y_2 and Y_3 featuring static nature. Out of relation (44) one obtains:

- if Y_1 and Y_2 are applied simultaneously,

$$P_{T,d} = \left(\frac{Y_1}{Y_{1,cr}}\right) + \left(\frac{Y_2}{Y_{2,cr}}\right)^{\alpha_2+1} = 0.8 + (0.6)^2 = 1.16$$

The organism died because $P_T > P_{cr}$ = 1.0;

- if the loads come in succession, first one applies Y_1 by shock and then Y_2 statically, the effect of Y_1 when one apply Y_2 is a static one,

$$P'_{T,d} = \left(\frac{Y_1}{Y_{1,cr}}\right)^{\alpha_1+1} + \left(\frac{Y_2}{Y_{2,cr}}\right)^{\alpha_2+1} = (0.8)^3 + (0.6)^2 = 0.872,$$

is less P_{cr} = 1.0 .

Out of relation (48) one may calculate the minimum time of medical treatment. For example, if $m_{cr}/v_m = 1200\,h$ and $\alpha_m = \alpha_3 = 0.5$

$$t_{min} = 1200\times(1-0.872)^{\frac{1}{1.5}} \approx 305\,h,$$

or minimum 13 days of medical treatment.

If the organism is treated before destructive loading action when $P_{T,d}$ = 1.16, it is useful to make sure

$$R(t) > 1 - P_{T,d} = 0.16.$$

The deterioration concept is fundamental in analyzing the lifetime issue; it was correlated to critical specific energy participation (10). Based on these two concepts a new approach was developed for the calculus of lifetime, applicable to engineering structures and for establishing the medical treatment to living organisms, taking into consideration the influence of deterioration.

The analysis showed that in the case of nonlinear behaviour the total effect is different from the sum of individual effects [37]. Consequently, it was necessary to develop a common theory for engineering structures and living organisms in the area of nonlinear effects superposition. This theory will allow in the future, in the case of living organisms, the prescription of medicines and the determination of a precise treatment program, based on mathematical correlations of the minimum treatment duration with a certain drug ((48) and (51)).

The general relation (19) correlated with relation (10) allows the deterioration dependent lifetime calculation for engineering structure, by taking into consideration the nonlinear matter behavior (21).

The general relation (12) allows the calculation of the residual or remaining lifetime. One deduced analytical relations for the accumulation of effects in the case of an engineering structure under fatigue loading (18), under fatigue loading in creep conditions (28) and in the case of a living organism undergoing the action of a pollutant, radiation flow and stress (41). From the calculation examples we have found the way to actually calculate the fatigue life for an engineering structure and the minimum time of medical treatment.

Nomenclature

C - constant of material in the laws of matter behavior;
D(t) - deterioration of matter after time *t*;
E, E_{cr} - specific energy; critical specific energy;
N - number of loading cycles to failure or fatigue life;
K_σ - stheoretical stress concentration factor;
Pcr -critical participation of the specific energy;
Pi; PT - specific energy participation of the i^{th} load; total participation of the specific energies;
$P(\sigma_a)$; $P(\sigma_m)$ - specific energy participation due to normal stress amplitude and due to mean normal stress, respectively;
S; S_{cr} - stress produced upon an organism and its critical value, respectively;
X - the effect of loading;
Y_i ; $Y_{i,cr}$ - load *i* and its critical value;
c_i - concentration of a pollutant *i* and its critical value, respectively;
k - exponent in the law of matter behavior;
t - time;
Φ; Φ_{cr} - radiation flow and its critical value, respectively;
γ_s - surface quality coefficient;
ε_d - dimensional coefficient;
σ, σ_{max}, σ_{min} - normal stress, maximum and minimum stress, respectively;
σ_a, σ_m - normal stress amplitude; mean normal stress;
σ_u - ultimate normal stress;
σ_{-1};$\sigma_{-1}(N)$ - fatigue limit (after N_0 cycles loading); sample fatigue strength after *N* cycles of loading;
$\sigma_{-1,s}(N)$ - fatigue strength of any part of a mechanical structure after *N* cycles of loading

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